Anoka-Hennepin Secondary Curriculum Unit Plan

Department:	Math	Course:	AP Calculus AB Test Prep/Enrichment	Unit 1 Title:	AP Review Topics	Grade Level(s):	12
Assessed Trimester:	Trimester A	Pacing:	9-10 days	Date Created:	04/27/2011	Last Revision Date:	06/16/2011

Course Understandings: *Students will understand that:*

A. The meaning of limit represents function behavior.
B. The meaning of the derivative represents a rate of change and is a local linear approximation and should understand that derivatives can be used to solve a variety of problems.
C. The meaning of the definite integral is a limit of Riemann sums and as the net accumulation of change and will understand that you can use integrals to solve a variety of problems.
D. The relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
E. You can model a written description of a physical situation with a function, a differential equation, or an integral.
F. You can use technology to help solve problems, experiment, interpret results, and support conclusions.

DESIRED RESULTS (Stage 1) - WHAT WE WANT STUDENT TO KNOW AND BE ABLE TO DO?

Established Goals

Minnesota State/Local/Technology Standard(s) addressed:

- **Advanced Placement (AP - AP CollegeBoard): Functions, Graphs, Limits, Derivatives, Integrals**
  a. **Analysis of graphs**
    - With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

  b. **b. Limits of functions (including one-sided limits)**
    - An intuitive understanding of the limiting process
    - Calculating limits using algebra
    - Estimating limits from graphs or tables of data

  c. **Asymptotic and unbounded behavior**
    - Understanding asymptotes in terms of graphical behavior
    - Describing asymptotic behavior in terms of limits involving infinity
    - Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

  d. **Continuity as a property of functions**
    - An intuitive understanding of continuity.

  e. **Concept of the derivative**
    - Derivative presented graphically, numerically, and analytically
    - Derivative interpreted as an instantaneous rate of change
    - Derivative defined as the limit of the difference quotient
    - Relationship between differentiability and continuity

  f. **Derivative at a point**
    - Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
    - Tangent line to a curve at a point and local linear approximation
    - Instantaneous rate of change as the limit of average rate of change
    - Approximate rate of change from graphs and tables of values

  g. **Derivative as a function**
    - Corresponding characteristics of graphs of f and f'
    - Relationship between the increasing and decreasing behavior of f and the sign of f'
    - The Mean Value Theorem and its geometric interpretation
    - Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

h. **Second derivatives**
  - Corresponding characteristics of the graphs of f, f', and f''
  - Relationship between the concavity of f and the sign of f''
  - Points of inflection as places where concavity changes

i. **Computation of derivatives**
  - Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
  - Derivative rules for sums, products, and quotients of functions
  - Chain rule and implicit differentiation

j. **Interpretations and properties of definite integrals**
  - Definite integral as a limit of Riemann sums
  - Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
  - Basic properties of definite integrals (examples include additivity and linearity)

k. **Applications of integrals**
  - Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

l. **Fundamental Theorem of Calculus**
  - Use of the Fundamental Theorem to evaluate definite integrals
  - Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

m. **Techniques of antidifferentiation**
  - Antiderivatives following directly from derivatives of basic functions
  - Antiderivatives by substitution of variables (including change of limits for definite integrals)

n. **Applications of antidifferentiation**
  - Finding specific antiderivatives using initial conditions, including applications to motion along a line
  - Solving separable differential equations and using them in modeling (including the study of the equation $y=ky$ and exponential growth)

o. **Numerical approximations to definite integrals**
  - Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Transfer

Students will be able to independently use their learning to: (product, high order reasoning)
- Create a slope field

Meaning

Unit Understanding(s):	Essential Question(s):
Students will understand that: ● Intuitively, the meaning of a limit ● How to evaluate a limit graphically, numerically, and algebraically ● The connections between limits and asymptotes ● The relationship between limits and continuity ● The difference between one-sided and two-sided limits The definition of a derivative as a limit ● The definition of a derivative as a slope of a tangent line ● How to evaluate a derivative graphically, numerically, and algebraically ● A derivative is the instantaneous rate of change ● The connection between integrals and area under a curve ● The integral is a limit of a Riemann sum ● The definition of a integral as an antiderivative ● The Fundamental Theorem of Calculus (Parts 1 and 2)	**Students will keep considering:** ● Is there a connection between a limit and a function's asymptotes? ● Can I find the limit by looking at the graph of the function? ● Can a function have a limit at a point of discontinuity? ● Can the limit approaching the *x*-value from the left be different from the limit approaching the same *x*-value from the right? ● Does the limit have to be a number? ● Do derivatives exist at all points? ● What does a derivative tell me? ● Is there some connection between a derivative and slope? ● Are derivatives used in any real-life situations? ● Are all functions integratable? ● What does an integral tell me?

● How to evaluate an integral graphically, numerically, and algebraically ● How integrals relate to net area and total area ● A slope field and differential equations	● Is there some connection between an integrals and area? ● Are integrals used in any real-life situations? ● What are all these dashes on this coordinate plane?

Acquisition

Knowledge - Students will: ● Limits ● Asymptotes ● Continuity/Discontinuity ● Definition of the Derivative ● Derivative Notation ● First and Second Derivative Test for Extrema ● Concavity Test ● Position, velocity, and acceleration ● Related Rates ● Normal and Tangent Lines ● Derivatives on a calculator ● Increasing/Decreasing functions ● Mean Value Theorem Integral Notation ● Definition of Integral ● Integral approximation methods ● FTOC (part 1 and 2) ● Slope Field ● Integrals on a calculator ● Area and Volume ● Integral Rules	**Reasoning - Students will:** ● Interpret how limits and asymptotes connect to one another. ● Analyze graphs using limits ● Determine when to use left and right-hand limits to classify discontinuities ● Determine when to use which differentiation rule ● Classify extrema using derivatives and sign charts ● Identify inflection points using second derivatives (concavity test) ● Analyze position, velocity, and acceleration ● Interpret the meaning of different rates of change ● Interpret the meaning of a tangent or normal line. ● Classify behavior of a graph using a derivative ● Understand the conditions for which the Mean Value Theorem applies ● Interpret the meaning of integral as accumulated areas ● Interpret a slope field as a general solution to a differential equation ● Determine appropriate integral/cross-section to calculate area and volume ● Interpret the integral of a rate as net change ● Determine when to use which integral rule **Skills - Students will:** ● AB1-1: Use limits to calculate vertical and horizontal asymptotes and end behavior ● AB1-2: Use limits to determine continuity and discontinuity ● AB1-3: Use one-sided limits to determine asymptotes & points of discontinuity ● AB2-1: Use differentiation rules and techniques to calculate derivatives ● AB2-2: Use derivatives to find position, velocity, and acceleration ● AB2-3: Use derivatives to solve related rates problems ● AB2-4: Write the equation for a tangent and/or a normal line to a curve ● AB2-5: Use derivatives to identify and classify extrema and inflection points ● AB2-6: Use derivatives to calculate the mean value of a function over an interval ● AB3-1: Use approximation methods and geometry to evaluate integrals ● AB3-2: Use a differential equation to model slope at points in the coordinate plane ● AB3-3: Use integrals to calculate area and volume ● AB3-4: Solve integral application problems ● AB3-5: Use integral rules and techniques to calculate integrals

Common Misunderstandings	Essential new vocabulary
• Students think that a limit cannot exist at a point of discontinuity • Students get confused with the left/right limit notation • Students have trouble with factoring higher degree polynomials • Students have trouble with complex fractions Students have trouble interpreting graphs of velocity and acceleration • Students have trouble with compound chain rule • Students have trouble with implicit differentiation • Students have trouble with related rate problems • Students have trouble with integrating vs find total area • Students have trouble with the "*u*" substitution method • Students have trouble with key sequences using their calculator • Students have trouble with volume problems when areas are revolved around something other than the *x*- or *y*-axis	• Due to review topics, there is no new vocabulary.